

# Computer displays in the teaching of physics

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## INTRODUCTION

An egg falls on the floor and breaks. Now run the movie backward: the broken egg reassembles itself and jumps into the air. Everyone laughs; they know that all the king's horses and all the king's men cannot put Humpty Dumpty together again. Even a child recognizes that an egg will not reassemble itself. *Why* won't it do so? The answer is deep, involving concepts of order, entropy, irreversibility. The intuition is simple and natural. The precise formulations should be embedded in a developed intuition.

Higher education, particularly in the sciences, tends to start with the formal, the symbolic, the abstract. Usually it is only after the equations and concepts are mastered that one is able to develop an intuition for the subject. And the teacher usually provides no help in the intuition-building process. Even in laboratory and demonstration experiments a long chain of inference lies between the observations and the physical questions at issue.

Our scientific education is largely backward, i.e., formalism first, intuition later (if at all). By teaching in this order we repel the majority of people who do not feel comfortable starting with equations. We also make it difficult for the professionals, since we offer them little assistance in developing the intuitive grasp of a subject from which new ideas spring.

The display computer provides a powerful means for modeling phenomena of nature. It can help the student visualize some effects more directly than through equations. Moreover, there are realms of physics in which one cannot hope to have a direct experience with the senses. For example, the high speeds of special relativity and the sub-microscopic scale of atomic physics closes both to direct visual observation. This paper deals with some attempts to present as visually and directly as possible some of the fundamental results of special relativity and quantum physics using computer displays.

## Relativity

### Rocket travel

Figure 1 shows a display tube face containing a view of a country road as might be seen from the front windshield of a "rocket car." Lining the road are telephone poles with crosspieces making the poles L-shaped. Because of computer memory limitations, only three poles are displayed on each side of the road. The operator controls the forward or backward acceleration of the rocket (the "proper acceleration") down the road by giving light pen commands. Rocket velocity is limited only by the laws of relativity and can approach the velocity of light. However, the visual presentation is *scaled*, so that relativistic effects are easily visible. This scaling can be interpreted in either one of two alternative ways: (1) the road is normal width, and the speed of light is sixty miles per hour, or (2) the speed of light has its normal value, while the telephone pole height and the road width are comparable to the dimensions of the planet Jupiter. Either interpretation applies to the one display.

By manipulating the display, an operator can become familiar with the following effects due to time delays for light propagation (effects 1 and 2 below) and due to the kinematics of spacetime (effects 3 thru 5 below).

#### 1. *Bending Poles.*

As poles move toward the observer ("forward motion down the road") the poles appear to be bent backward (Figure 2), with the tops farther back down the road than the middles. This is due to the fact that the top of a pole is farther from the observer's eye than the middle of the pole. Therefore it takes light longer to reach the observer's eye from the pole top than from the pole middle. Therefore the observer sees the more distant top of the approaching pole where it was at an earlier time (i.e., farther away from him) than the observed position of the middle portion of the pole.

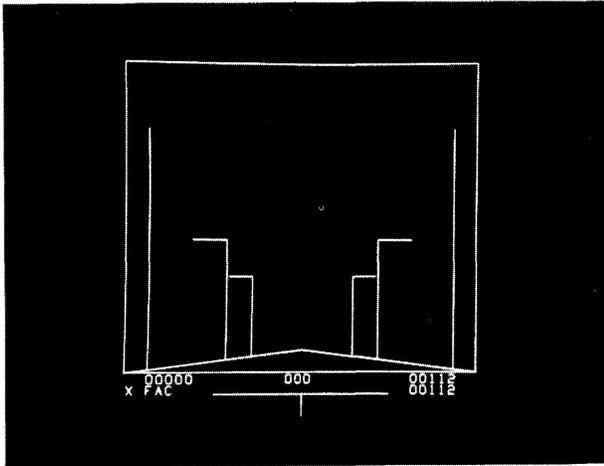


Figure 1—Perspective view of “country road” as seen through front windshield of rocket ship at rest. Three L-shaped telephone poles are shown on either side of the road. Numbers displayed below picture: center number is rocket speed  $v/c$  along road as a fraction of the speed of light; on the right, lower number is reading of clock carried in cockpit, upper number is time read off adjacent road clock; number at left is count of telephone poles passed. Light pen controls: operator sets forward or backward acceleration on horizontal line at bottom of picture; letters at left: X means “start again”; F means “freeze”; C means “coast without acceleration”; A means “go back to accelerating.”

2. Terrell rotation effect.

The cross pieces of the poles appear to rotate about the pole as axis as the poles approach the observer (Figure 2). This effect is a rather-more-complicated result than (1) of the relative time delay of signals from different parts of the pole. The effect was analyzed by Terrell<sup>1</sup> in 1959, more than fifty years after

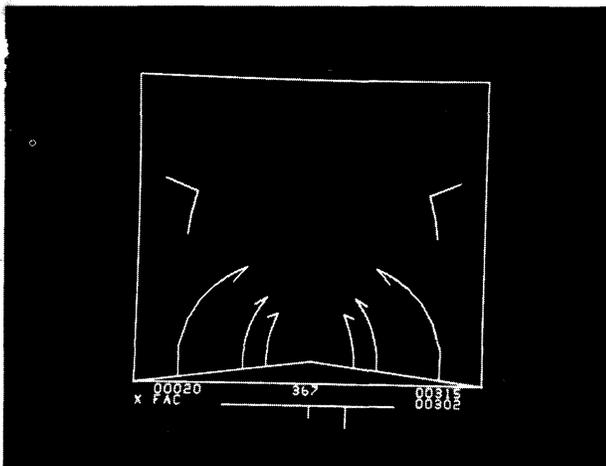


FIGURE 2—Rocket moves down the road with speed  $v/c = 0.367$ . Acceleration is positive. Poles appear to bend backward down the road; crosspieces show Terrell Rotation Effect.

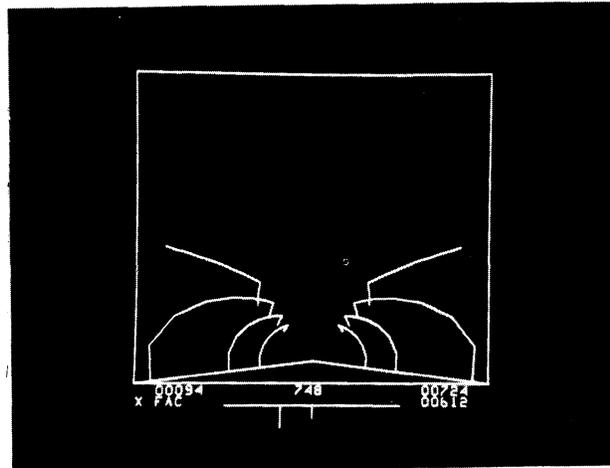


FIGURE 3—Rocket moves at velocity almost three-quarters of the speed of light. Acceleration is negative. Note different readings on rocket clock (lower number on right) and passing road clocks (upper number on right)—the “time dilation effect.”

relativity was propounded. This effect results automatically from the transformation equations and was not specially programmed.

3. Time dilation.

The lower number of the pair on the right of Figure 3 is the time measured in the cockpit of the rocket car, in arbitrary units (about three units per second). The number above is the time, in the same units, read off a series of road clocks, synchronized in the road frame, as each in turn passes by the rocket. The rocket clock always shows a lower cumulative reading than the adjacent road clock. By taking a round trip down the road and back while watching the pole counter at the left of

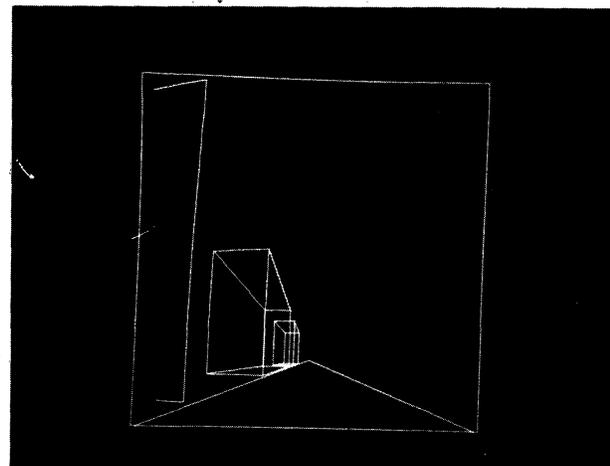


FIGURE 4—Display of cubes along one side of road; rocket at rest.

the figure, the operator can see the two clock readings to demonstrate the "twin paradox" according to which a traveling twin who moves at high velocity is younger on his return than his stay-at-home-brother.

#### 4. *Non-addition of velocities.*

No matter how high a rocket acceleration is set by the operator, the speed of the vehicle (expressed by the central numeral as a decimal fraction  $v/c$  of the speed of light) cannot exceed that of light. Instead, the speed with respect to the road is seen to increase more and more slowly as the velocity  $v/c$  approaches unity. This is true even though the acceleration of the rocket maintains its constant value ("constant proper acceleration"). In a real rocket, the passengers would continue to be pressed against the backs of their seats with a constant pressure—a pressure not reproducible with a computer display! One way to describe this effect is to say that velocities do not add: the second-by-second increase of rocket velocity in the local frame is not equal to the incremental change of velocity of the rocket in the road frame.

#### 5. *Surprising changes in aberration angle.*

When the rocket is first accelerated at a high rate from rest *forward* along the road, the poles appear to move initially *away* from the observer instead of toward him as one expects from everyday experience. This result was unanticipated and was initially perplexing to those who developed the display. It is explained in terms of aberration angle: At high velocity, objects are observed in directions different from those at which they would be seen if the observer were at rest. If the acceleration is great enough, the velocity change modifies the aberration angle to a nearby pole faster than displace-

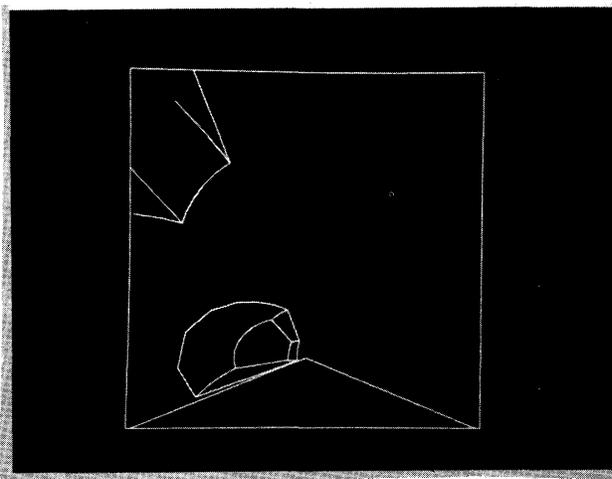


FIGURE 5—View of cubes at high rocket velocity. Leading face of distant cube is turned around, so we look at its back or inner side.

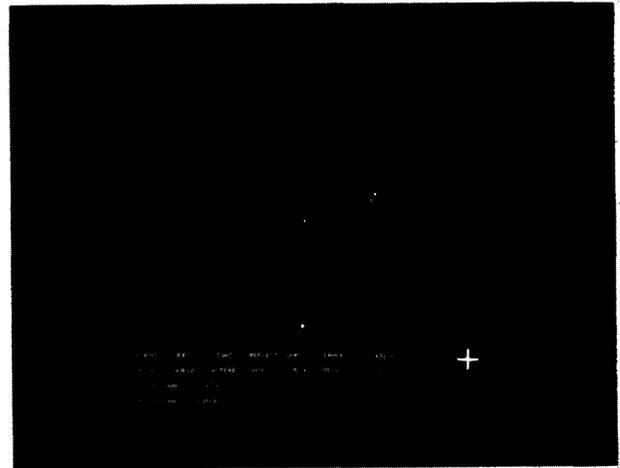


FIGURE 6—Spacetime diagram display. Horizontal axis represents a single space dimension; vertical axis represents time dimension. Point A shows location of event in spacetime (coordinates of A displayed on lower right in identical units, such as meters). To place a point-event, tracking cross on right is moved by light pen to desired location and button EVENT touched to create labeled point. This diagram is drawn for the "original" reference frame, so speed  $v/c = 0$  on lower left.

ment down the road can displace the pole in the opposite direction.

Figures 4 and 5 show a similar display with cubes on one side of the road instead of telephone poles on both sides. The cube display is destined for film loop use and not as an interactive program.

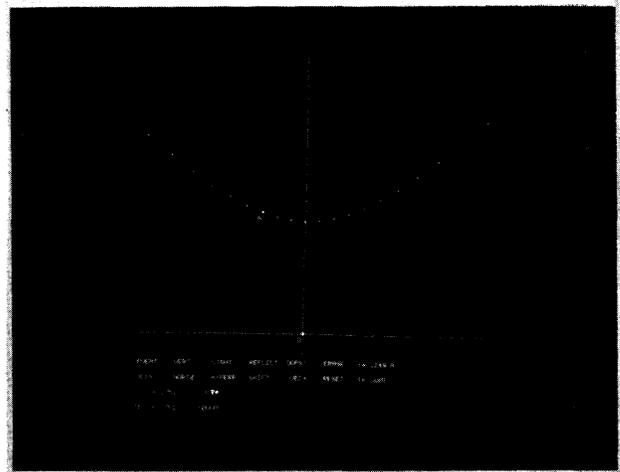


FIGURE 7—Spacetime diagram locating event A of preceding figure for rocket observer moving at speed  $v/c = 0.751$  relative to "original" frame. Transformation is accomplished by holding light button + on symbol -LT+. Note altered space and time coordinates as observed in rocket frame. "Invariant hyperbola," along which event A slides during transformation, has been added using HYPERB button.

We must not leave these relativity pictures without acknowledging that they are deficient in at least two respects: at high velocities the *colors* and *intensities* of observed objects would vary rapidly as they approach and pass the observer. These changes in color and intensity are not reproduced in the display programs presented above.

**Spacetime diagram**

A more analytic manipulation of the results of special relativity is presented in the spacetime diagram display (Figure 6). This display demonstrates how the *x* (space) and *t* (time) coordinates of an event as observed in one frame of reference are related to the corresponding *x'* and *t'* coordinates of the same event as observed in a second frame of reference in high uniform motion with respect to the first frame. The horizontal axis in Figure 6 is the *x*-axis. The vertical axis is the *t* (time) axis expressed in units of length by plotting *ct*. The location of a point in the *x-t* plane (such as point A) gives the space and time coordinates of that event (see coordinate reading for point A in lower right portion of diagram). Coordinates of events in all frames are measured with respect to some common agreed-upon "reference event" (or "starting gun") taken by definition to occur at the zero of time and space (event 0 at the origin of Figures 5 and 7).

Figure 7 shows the same event A and its coordinates as measured with respect to a rocket reference frame moving at a speed  $v/c = 0.751$  along the *x* spatial direction with respect to the original reference frame (value  $v/c$  recorded automatically in lower left corner of display in Figure 7). The transformation of the coordinate

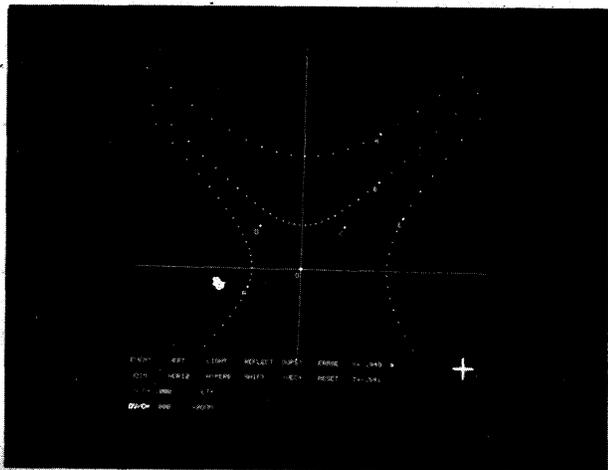


FIGURE 8—A field of point-events, showing some of the constructions available on the display. Light cone through origin event 0 divides spacetime into three kinds of regions with respect to this origin event.

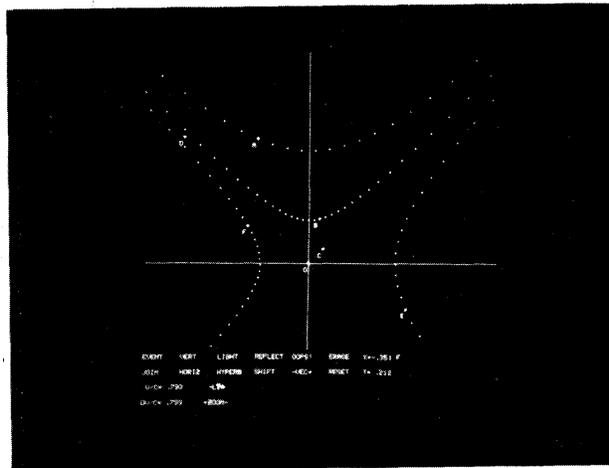


FIGURE 9—Diagram of Figure 8 transformed to rocket frame. These two figures, taken together, illustrate the Doppler shift, time dilation, relativity of simultaneity, relative synchronization of clocks, and invariance of the spacetime interval.

plot is accomplished by touching with the light pen the "Lorentz transformation button" (LT+) on the display. The relative velocity between frames increases continually as long as the "button" is touched with the light pen. The reverse transformation back to coordinates in the original reference frame is accomplished by pushing the *minus* sign in the symbol "—LT+".

Event-points are placed on the display by using the light pen to move the tracking cross to the desired place on the screen and then touching the "EVENT" button. Up to 16 points can be displayed on the screen simultaneously (Figure 8). Each new point is automatically labeled with a letter in alphabetical order. Coordinates of any single event are displayed when the corresponding display point is touched by the light pen. When a transformation is made from one frame to another, the entire field of points is simultaneously transformed, each to its new coordinate position (Figure 9).

Other light buttons are used to erase selectively (ERASE), to clean the slate (RESET), to contract and expand the scale of the display (ZOOM), to join events with lines (JOIN), to draw through any event: an invariant hyperbola (HYPERB) or crossed straight lines at 45 degrees from horizontal (LIGHT) or a horizontal line (HORIZ) or a vertical line (VERT). The respective space and time coordinates of two events can be added to create a third event (ADD).

Using these commands, one can investigate a wide variety of the properties of spacetime including:  
 the world lines of particles and light flashes—their paths through spacetime  
 invariance of the interval  $(ct)^2 - x^2$  that separates two events

the light cone as a partition in spacetime, (invariance of the speed of light.)

regions of spacetime: timelike, lightlike and spacelike relations between events

clocks using light pulses

Doppler shift

time dilation

Lorentz contraction

non-additivity of velocities

The components of momentum and the relativistic energy of a particle transform from one frame to another in the same way that the space coordinates and time of an event transform. This means that a point on the spacetime diagram can be used to represent the x-momentum (horizontal dimension) and energy (vertical dimension) of a particle. Using this interpretation of the diagram, the operator can analyze one-dimensional collisions between particles. In particular the REFLECT button reflects all points about the vertical axis. This can be used to represent the exchange of x-momentum of two particles in an elastic collision as observed in the center-of-momentum frame.

The spacetime diagram is especially versatile in that the student can both manipulate it to get a qualitative feel for relativistic phenomena and also read numbers

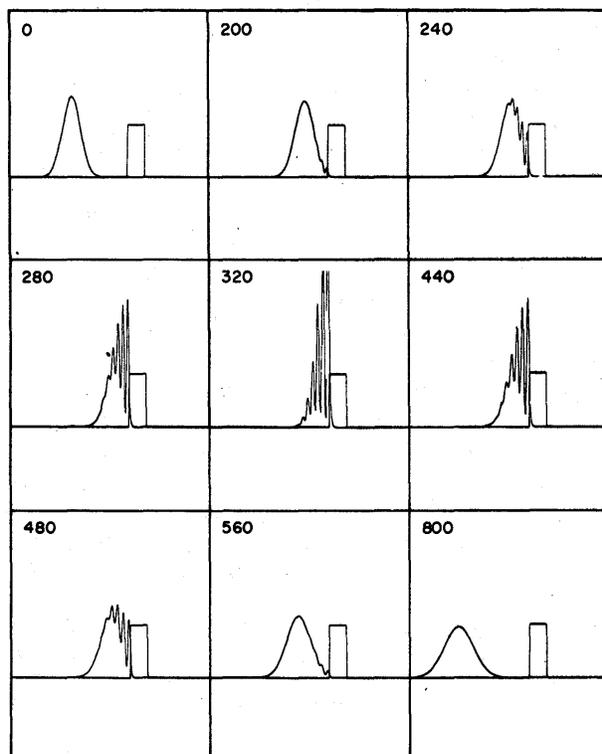


FIGURE 10—Gaussian wave packet scattering from a square barrier. The average energy is *one half* the barrier height. Numbers denote the time of each configuration in arbitrary units.

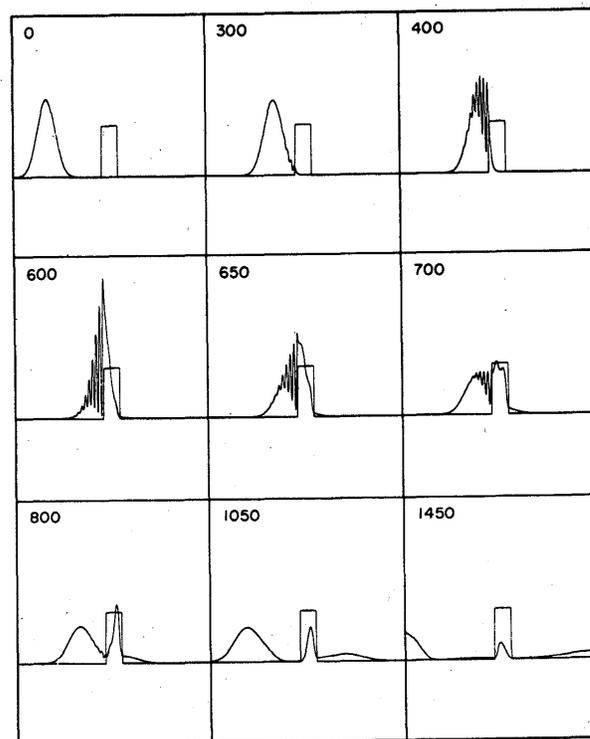


FIGURE 11—Gaussian wave packet scattering from a square barrier. The average energy is *equal* to the barrier height. Numbers denote the time of each configuration in arbitrary units.

from the figure accurate to three places. In this way he can pass easily between the analytic and intuitive aspects of learning relativity.

### Quantum physics

#### One-dimensional scattering

In a quantum mechanical description of nature it is impossible in principle to describe the instantaneous position of a moving particle with complete precision. The dynamical theory provides a probability distribution as a function of position that evolves in time.\* This is to be understood in terms of the real world as the cumulative result of a succession of a large number of independent identical experiments, each one of which by itself is unpredictable. One can detect both classical

\*The equation to be solved is the time dependent Schrodinger equation. In general we solve this partial differential equation in position space for the complex function  $\psi(x,t)$  at each time  $t$  for somewhere between 1000 and 2000 values of  $x$ . Except where otherwise indicated the absolute square of this function is the plotted curve in the figures. Interested readers may find the computational techniques described in Am. Jour. of Phys. *35*, 177 (1967).

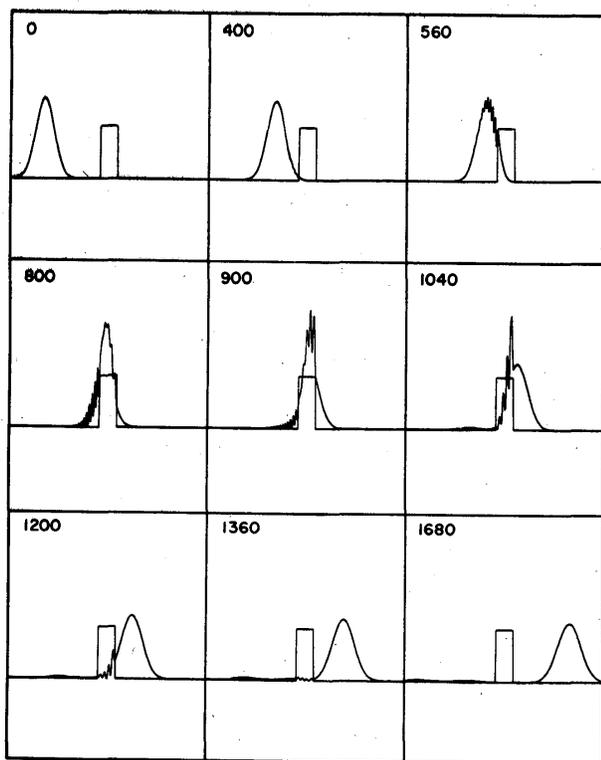


FIGURE 12—Gaussian wave packet scattering from a square barrier. The average energy is *twice* the barrier height. Numbers denote the time of each configuration in arbitrary units.

and non-classical patterns of behavior in these situations. For example in the first scene of the film entitled “Barriers (see Figure 10), one observes the probability distribution for a particle that is approaching a barrier with an energy that is not sufficient to allow it to be transmitted. During the most violent part of the scattering the probability distribution penetrates the region of the barrier. This corresponds to a finite probability at that time of finding the particle in a region that is inaccessible to it in a classical description of nature. On the other hand, toward the close of the first scene one sees the wave packet reflected from the region of the barrier in much the same way that a classical particle would bounce off a wall. The viewer will note, however, that the probability distribution has widened substantially since the beginning of the process. This, too, is strictly quantum effect.

The second scene in this film (see Figure 11) shows the quantum description of a particle approaching a barrier with energy just equal to the barrier height. In this case one would expect on the basis of classical physics that the particle would enter the region of the potential and lose kinetic energy until finally all its kinetic energy is expended in mounting the barrier, where it would

then find itself in a region of no force with all its kinetic energy gone and, therefore, would remain in the region of the barrier forever. The film shows the quantum mechanical analog of this special case. Sure enough, a probability packet remains in the region of the barrier, at least for a long time. In addition, however, one observes transmitted and reflected packets representing the chances for transmission and reflection when many experiments are carried out. This “splitting up of the incident packet” due to the fundamentally statistical character of the quantum picture is foreign to classical mechanics.

In the final scene (see Figure 12) the particle has more than enough energy to overcome the barrier, but we find, nonetheless, a small but significant probability that the particle will be reflected.

Thus far we have displayed the time development of probability in *space*—the so-called “configuration space representation” of events. An equivalent method for describing an encounter of a particle with a barrier is the “momentum space representation”—that is, in terms of the momentum distribution of the probability packet. Quantum mechanics shows that the momentum space representation is every bit as good as the configuration space representation; each one carries all the information obtainable about the probable results of experiment.\*\*

Most students have their first exposure to quantum mechanics in configuration space and, as a result, obtain a somewhat lopsided view of quantum mechanics. The next film, entitled “Momentum Space” portrays a collision as described by quantum mechanics in configuration space *and* in momentum space, in order to contrast the two and to see how the same physical event manifests itself in different representations. The first scene depicts the evolution in time of the position probability distribution of a particle as it approaches a region in which it will feel an attractive force (a potential well). After the “event” we observe a substantial probability of reflection. The second scene in this film depicts the evolution in time of the momentum probability distribution. Momenta corresponding to motion to the right in position space are here plotted to the right of the center line and momenta corresponding to leftward motion in position space are plotted to the left.†

\*\*The computer displays filmed here present squared magnitudes of momentum and configuration functions. Information about complex phase (necessary to demonstrate the complete equivalence of momentum and configuration representations) is thus masked.

†The function here plotted is the absolute square of the Fourier transform of the solution to the dependent Schrodinger equation in position space. Only the recent availability of the Cooley-Tukey algorithm for Fourier Transforms as a standard library subroutine makes this computation a feasible one.

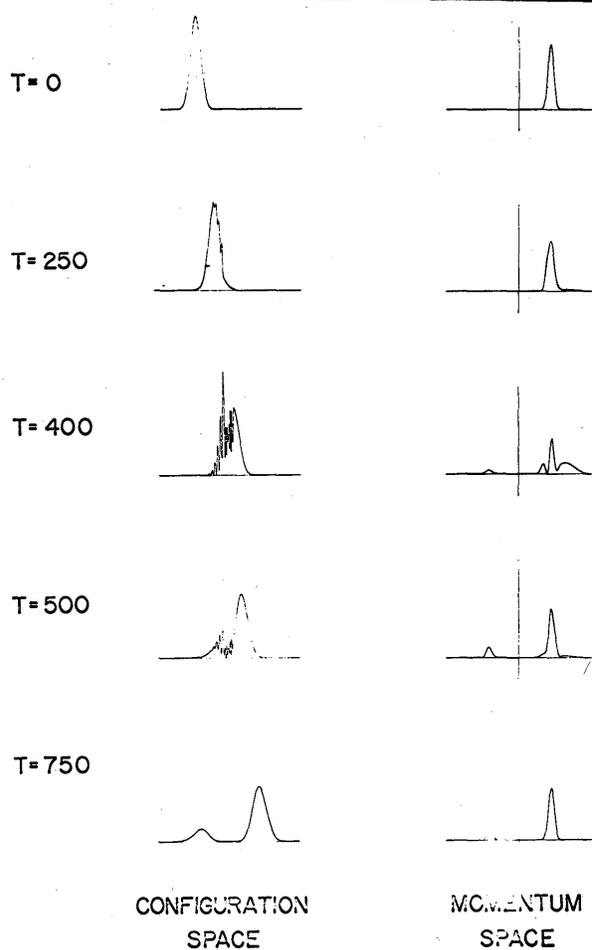


FIGURE 13—The probability distributions in configuration space and momentum space at corresponding instants of time for the scattering of a Gaussian wave packet from a square well. The particle energy is equal to one half the magnitude of the well depth. The vertical scales are arbitrary. See A. Goldberg, H. M. Schey and J. L. Schwartz, *American Journal of Physics*, 36, 454, (1968).

As long as the particle travels freely toward the region of the force the distribution of momenta remains unaltered, an aspect of quantum theory that Newton would have been content with. As the particle begins to feel the effect of the force, the distribution of momenta undergoes severe distortions, due in part to the increased kinetic energy (and therefore increased momentum) as the packet passes over the potential well. After a short while the momentum distribution begins to develop a peak centered about some average leftward momentum, the momentum of the reflected packet. When the interaction between packet and well is completed, the momentum distribution again remains constant while the probability packets in configuration space sail majestically offstage. Figure 13 shows these two descriptions of the collision at five instants of time.

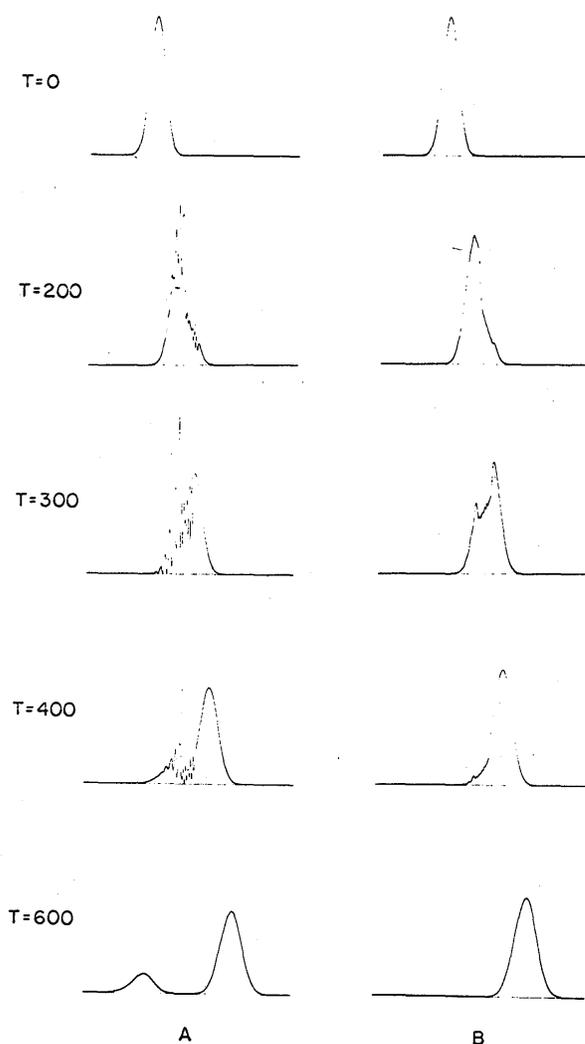


FIGURE 14—The position probability density at selected times for scattering from a square well and a Wood-Saxon well. See A. Goldberg, H. M. Schey, J. L. Schwartz, *American Journal of Physics*, 35, 777, (1967).

The barriers encountered in real quantum collisions are atoms and nuclei. These potential barriers and wells are not square in shape but have a potential that changes smoothly with position. In this respect the quantum displays shown thus far do not correspond to reality. Happily, the computer makes its calculations using a point-by-point specification of the potential. Therefore interaction with a potential with "rounded corners" is no more difficult to program and compute than an interaction with a square potential. The result of rounding the corners of the barrier is a dramatic reduction in the probability of reflection from the well. In analogy to electromagnetic wave phenomena, one may say a more gradual "change in impedance" at the sur-

face of the well results in less reflection of the incident "wave."

The effects of softening the barrier walls is presented in the next film, entitled "Edge Effects." The results of progressive rounding of the corners is shown in three stages. Notice that during interaction the structure of the probability is much smoother for the rounded barrier than for the square one, also that the probability for reflection is less for the rounded barrier. Figure 14 contrasts the scattering from a square well with the scattering from a well which has a diffuse surface.

### CONCLUSION

It is not very difficult to propose additional aspects of modern science that are difficult to visualize and for which this kind of presentation would be of some benefit. In fact, under development now at the Computer Film Project of the MIT Education Research Center are efforts in physical chemistry, electrodynamics, and molecular biology. Each of these, we have reason to believe, will enhance the understanding of students as well as broaden the range of tools available to the instructor.

Some of the displays discussed here have been used informally with students and professors. Others have recently been made available commercially as film loops. As yet the use has not been widespread enough to allow us to draw firm conclusions about their effectiveness in developing intuition about relativity and quantum physics. What little evidence we have leads us to want to continue developing these materials. Students appreciate the opportunity to focus on the phenomena without being burdened by the constant onslaught of mathematical symbols. Some of our most gratifying moments occur when professional physicists of the highest caliber respond to one of these films with wonder and delight and indicate that, although they have grown up with the field, they have never seen the behavior of the system revealed this clearly.

### REFERENCES

- 1 J TERRELL  
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