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CRITERIA FOR A POWER PRODUCING THERMONUCLEAR REACTOR WITH ARBITRARY LOSSES

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Lawson (1957) has considered an idealized controlled thermonuclear reactor and has shown that for a net power gain it is necessary that $(nt)_{DD} > 10^{16}$ for a deuterium gas, and $(nt)_{DT} > 10^{14}$ for a deuterium-tritium gas. In Lawson's model the gas is heated to a uniform temperature which is maintained constant for a time t; also the density n of the ions is uniform and remains approximately constant for a small percentage burn-up. Lawson's criteria are necessary but not sufficient for it is assumed that the only energy loss is bremsstrahlung emission. All other energy losses, such as impurity and cyclotron radiation and various diffusion processes, are neglected. The energy losses outside the gas, as for example the Joule dissipation in the conductors which produce the confining electromagnetic field, are also neglected. Attempts to take into account such losses (Jensen et al 1958, Imoto 1959) inevitably lead to criteria which are less general and which depend upon the detailed nature of the reactor. We shall show however that whatever the losses are in a reactor, Lawson's criteria are sufficient provided the time t is interpreted in a slightly different way.

We first consider a highly idealized reactor in which the only energy expended is that necessary to heat the gas at constant volume. At time t = 0 an atom (in this case a hydrogen isotope) is raised to an energy of 3kT (which is shared by the ion and the electron), where T is the temperature, and at time t it strikes the wall and its energy drops back to zero. Let

$$P_r = \alpha n \langle \sigma v \rangle \qquad \begin{cases} \alpha_{DD} = 1.0 \ \ 10^{-5} \\ \alpha_{DT} = 7.0 \ \ 10^{-6} \end{cases} \tag{1}$$

be the reaction power per ion (assuming that tritium formed in the D-D reaction is recycled, and that there are equal atomic concentrations in the D-T reaction). If all reaction products escape the energy delivered to the walls as heat is $3kT + P_r t$ per ion in time t. If η is the efficiency of converting this heat into useful energy, there is a net energy gain when

$$\eta(R + 1) > 1 \tag{2}$$

where $R = P_r t/3kT$. Or,

$$R = nt \frac{\alpha}{3k} \frac{\langle \sigma v \rangle}{T} > \eta^{-1} - 1 \simeq 2 \tag{3}$$

for an efficiency of approximately 1/3.

In (3) nt has its minimum value at

$$\frac{d \log \langle \sigma v \rangle}{d \log T} = 1 \qquad (4)$$

and using the log $\langle \sigma v \rangle$ - log T curves (Thompson 1957, Jensen et al 1958) we find that their slope is unity at $T_{DD} = 140$ keV, $\langle \sigma v \rangle_{DD} = 7.0 \; 10^{-17}$, and $T_{DT} = 25$ keV, $\langle \sigma v \rangle_{DT} = 5.5 \; 10^{-16}$. Hence, using (3), it follows

$$(nt)_{DD} > 2.10^{15}, \quad (nt)_{DT} > 6.10^{13}, \qquad (5a,b)$$

for a net energy gain. In these criteria t is either the life-time of an individual ion in a continuously running reactor, or is the duration of the discharge in a pulsed reactor which has no diffusion losses. At the temperatures $T_{DD} = 140$ keV, $T_{DT} = 25$ keV the bremsstrahlung power loss is small compared with P_r and (5a,b) are also essentially the correct values for Lawson's model. In fact, the criteria (5a,b) are generally true for any reactor in which the energy loss per ion in the ion life-time is small compared with 3kT.

We consider now a more realistic reactor which has a variety of energy losses. Let

$$Q = \frac{\text{total energy per ion supplied to reactor in time } t}{3kT}$$

$$S = \frac{\text{energy per ion lost from gas to walls in time } t}{3kT}$$

Thus, Q is the total energy supplied to the reactor, of which R is the loss which can be converted to useful energy. There is now a net power gain when

$$\eta (R + S) > Q \qquad (6)$$

where R has the same meaning as before. For $\eta = \frac{1}{3}$,

$$R > 3Q - S,$$

and since $Q > S$, a sufficient condition is

$$R = \frac{P_r t}{3kT} > 3Q \qquad (7)$$

(A necessary condition is $R > 2Q$). The sufficient conditions for a net energy gain are therefore

$$(nt)_{DD} > 3\ 10^{15} Q, \quad (nt)_{DT} > 9\ 10^{13} Q \qquad (8a,b)$$

The total energy supplied to the reactor is approximately a linear function of time, and can be expressed in the form

$$Q = \alpha + \frac{t}{\tau} \qquad (9)$$

where $3kT\alpha$ is the initial energy expended in raising an ion to an energy of $3kT$, and τ is the time taken to supply the reactor with an energy of $3kT$ per ion. In the idealistic model considered previously, $\alpha = 1$ and $\tau = \infty$. There is little loss of generality if α is set equal to unity, and by combining (8a,b) and (9), it follows

$$\left(n \frac{\tau t}{\tau + t}\right)_{DD} > 3\ 10^{15}, \quad \left(n \frac{\tau t}{\tau + t}\right)_{DT} > 9\ 10^{13} \qquad (10a,b)$$

When $t \ll \tau$, we recover (5a,b) which are essentially Lawson's original criteria. However, when $t \gg \tau$, the sufficient conditions for a net energy gain are

$$(n\tau)_{DD} > 3\ 10^{15}, \quad (n\tau)_{DT} > 9\ 10^{13} \qquad (11a,b)$$

where τ is now the time taken by the reactor to consume an energy of $3kT$ per ion, or an energy of $3NkT$, where N is the total number of ions.

The conditions (10a,b) are for a reactor of arbitrary energy losses and are therefore quite general. It is interesting to notice that they are the same as the Lawson criteria with the exception that the time t is replaced with a 'reduced time' of $\tau t/(\tau + t)$. In a realistic model of a magnetically confined gas it seems not unlikely that the total energy consumed per ion will be large compared with $3kT$, and the appropriate conditions are therefore (11a,b). This means, for example, that in a gas of $n = 10^{16}$ cm^{-3}, $\tau_{DD} > 0.3$ sec, and $\tau_{DT} > 9\ 10^{-3}$ sec, or the energy expended in a D-D reactor must not exceed 3×140 keV/0.3 $\simeq$ 1 MeV per ion per sec, and in a D-T reactor must not exceed 3×25 keV/9 $10^{-3} \simeq$ 10 MeV per ion per sec. Whether or not these conditions are possible must depend on a detailed assessment of the losses peculiar to each type of reactor.

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